Extra Notes for Section 2.3:

In the expansion of \((1 + x)^n\), two times of the coefficient of \(x^5\) is the sum of the coefficient of \(x^4\) and \(x^6\). Find the value(s) of \(n\).

Answer

The \((r+1)^{th}\) term of the expansion = \(C_r^n (1)^{n-r} (x)^r = C_r^n x^r\).

If \(x^5 = x^r\), then \(r = 5\). If \(x^4 = x^r\), then \(r = 4\). If \(x^6 = x^r\), then \(r = 6\).

So, \(2C_5^n = (C_4^n + C_6^n)\).

\[
\frac{n! (2)}{(n-5)! 5!} = \frac{n!}{(n-4)! 4!} + \frac{n!}{(n-6)! 6!}
\]

\[
\frac{n! (2)}{(n-5)! 5!} = \frac{n!(6 \times 5)}{(n-4)! 6!} + \frac{n!(n-4)(n-5)}{(n-4)! 6!}
\]

\[
\frac{n! (2)}{(n-5)! 5!} = \frac{n!}{(n-4)! 6!} (30 + (n-4)(n-5))
\]

\[
12(n-4) = (30 + (n-4)(n-5))
\]

\[
12n - 48 = 30 + n^2 - 9n + 20
\]

\[
0 = n^2 - 21n + 98
\]

\(n = 7\) or \(n = 14\)